

UCLA Math 115A: Linear Algebra

Lecture, three hours; discussion, two hours. Requisite: course 33A.

Techniques of proof, abstract vector spaces, linear transformations, and matrices; determinants; inner product spaces; eigenvector theory.

P/NP or letter grading.

Math 115A is a core mathematics course required of all the various mathematics majors. The course material can be regarded as an elaboration of the linear algebra already covered in Math 33A. However, the level of abstraction and the emphasis on proof technique make this a difficult course for many students. Successful students emerge from the experience not only with a better understanding of linear algebra, but also with a higher level of mathematical maturity, better equipped to deal with abstract concepts.

The material covered in Math 115A includes linear independence, bases, orthogonality, the Gram-Schmidt process, linear transformations, eigenvalues and eigenvectors, and diagonalization of matrices. These topics are all covered in Math 33A though only in the context of Euclidean space. Topics in Math 115A that go beyond Math 33A include inner product spaces, adjoint transformations, and the spectral decomposition theorem for self-adjoint operators.

Three or four sections of Math 115A are offered each term. Also, an honors version Math 115AH runs parallel to Math 115A in some quarters. The content of Math 115AH is as follows:

Vector spaces, subspaces, basis and dimension, linear transformations and matrices, rank and nullity, change of basis and similarity of matrices, inner product spaces, orthogonality and, orthonormality, Gram-Schmidt process, adjoints of linear transformations and dual spaces, quadratic forms and symmetric matrices, orthogonal and unitary matrices, diagonalization of hermitian and symmetric matrices, eigenvectors and eigenvalues, and their computation, exponentiation of matrices and application to differential equations, least squares problems, trace, determinant, canonical forms. Systems of linear equations: solvability criteria, Gaussian elimination, row-reduced form, LU decomposition.

Textbook(s)

S. Friedberg, et al, *Linear Algebra*, 5th Ed., Pearson.

TOPICS

1.2 Vector Spaces over a Field

1.3 Subspaces

1.4, 1.5 Linear Combinations and Systems of Linear Equations; Linear Dependence and Linear Independence

1.5, 1.6 Linear Dependence and Linear Independence; Bases and Dimensions

1.6 Bases and Dimensions

1.6 Bases and Dimensions

2.1 Linear Transformations, Null Spaces, and Ranges

2.1 Linear Transformations, Null Spaces, and Ranges

2.1, 2.2 Linear Transformations, Null Spaces, and Ranges; The Matrix Representation of a Linear Transformation

. Midterm #1

2.2 The Matrix Representation of a Linear Transformation

2.3 Composition of Linear Transformations and Matrix Multiplication

TOPICS

2.4 Invertibility and Isomorphisms

2.4, 2.5 Invertibility and Isomorphisms; The Change of Coordinate Matrix

2.5 The Change of Coordinate Matrix

4.4 Summary – Important Facts about Determinants

5.1 Eigenvalues and Eigenvectors

5.1 Eigenvalues and Eigenvectors

5.2 Diagonalizability

5.2 Diagonalizability

5.2 Diagonalizability

. Midterm #2

6.1 Inner Products and Norms

6.1, 6.2 Inner Products and Norms; The Gram-Schmidt Orthogonalization Process and Orthogonal Complements

TOPICS

- 6.2 The Gram-Schmidt Orthogonalization Process and Orthogonal Complements
- 6.3 The Adjoint of a Linear Operator
- 6.4 Normal and Self-Adjoint Operators
- 6.4 Normal and Self-Adjoint Operators
- . Catch-up, Review